[C056/SQP182]

Advanced Higher Mathematics

Specimen Solutions

NATIONAL QUALIFICATIONS

SCOTTISH QUALIFICATIONS AUTHORITY

Section A (Mathematics 1 and 2)

A1. (*a*) $\dfrac{4}{x^2-4} = \dfrac{4}{(x-2)(x+2)} = \dfrac{A}{x-2} + \dfrac{B}{x+2}$

$= \dfrac{1}{x-2} - \dfrac{1}{x+2}$ **[2]**

(*b*) $\displaystyle\int \frac{x^2}{x^2-4}\,dx = \int 1 + \frac{4}{x^2-4}\,dx$

$\displaystyle = \int 1 + \frac{1}{x-2} - \frac{1}{x+2}\,dx$

$= x + \ln(x-2) - \ln(x+2) + c$ **[4]**

A2. (*a*) $a = 8 + 10t - \frac{3}{4}t^2$

$v = \int 8 + 10t - \frac{3}{4}t^2 dt$

$= 8t + 5t^2 - \frac{1}{4}t^3 + c$

$t = 0;\ v = 0 \Rightarrow c = 0$

$v = 8t + 5t^2 - \frac{1}{4}t^3$ **[2]**

(*b*) $s = \int v\,dt = 4t^2 + \frac{5}{3}t^3 - \frac{1}{16}t^4 + c'$

$t = 0;\ s = 0 \Rightarrow c' = 0$

$\therefore$ when $t = 10,\ s = 400 + \dfrac{5000}{3} - 625 = 1441\dfrac{2}{3}$ **[3]**

A3. $\int_0^2 \frac{x+1}{\sqrt{16-x^2}}dx$ | $x = 4\sin t$

$= \int_0^{\pi/6} \frac{4\sin t + 1}{\sqrt{16 - 16\sin^2 t}} 4\cos t\, dt$ | $\Rightarrow \frac{dx}{dt} = 4\cos t$

$x = 0 \Rightarrow t = 0;$

$= \int_0^{\pi/6} \frac{(4\sin t + 1) \times 4\cos t}{4\cos t}\, dt$ | $x = 2 \Rightarrow t = \frac{\pi}{6}$

$= \int_0^{\pi/6} (4\sin t + 1)dt$

$= [-4\cos t + t]_0^{\pi/6} = 2\sqrt{3} + 4 + \frac{\pi}{6} \approx 1\cdot059$ **[5]**

A4.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & -1\cdot1 \\ 1 & 3 & 2 & 0\cdot9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & -1\cdot1 \\ 0 & 2 & 1 & 0\cdot9 \end{array}\right) \quad \begin{array}{l} \\ (r_2' = r_2 - 2r_1) \\ (r_3' = r_3 - r_1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & -1\cdot1 \\ 0 & 0 & 1 & 0\cdot5 \end{array}\right) \quad \begin{array}{l} \\ \\ (r_3'' = 3r_3 + 2r_2) \end{array}$$

Hence $z = 0\cdot5$; $y = (1\cdot1 - 0\cdot5)/3 = 0\cdot2$;
$x = -0\cdot2 - 0\cdot5 = -0\cdot7$ **[5]**

Marks

A5. (*a*) $x^2 + xy + y^2 = 1$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$$ **[2]**

(*b*) (i) $x = 2t + 1;\quad y = 2t(t-1)$

$$\frac{dx}{dt} = 2;\ \frac{dy}{dt} = 4t - 2 \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2t - 1$$ **[2]**

(ii) $t = \frac{1}{2}(x-1) \qquad y = (x-1)\left[\frac{1}{2}(x-1)-1\right]$

$$= \frac{1}{2}(x-1)(x-3)$$ **[1]**

A6. (*a*)

$$u_3 = 2d + u_1 = 5$$
$$2d = 5 - 45$$
$$d = -20$$
$$u_{11} = 45 + 10(-20)$$
$$= -155$$ **[2]**

(*b*) $45r^2 = 5$

$r = \frac{1}{3}$ since v_1, ... are positive

$$S = \frac{45}{1-\frac{1}{3}} = 67\tfrac{1}{2}$$ **[3]**

A7. $n = 1$ LHS $= 1 \times 2 = 2$

RHS $= \frac{1}{3} \times 1 \times 2 \times 3 = 2$

True for $n = 1$.

Assume true for n and consider

$$\sum_{r=1}^{n+1} r(r+1) = \sum_{r=1}^{n} r(r+1) + (n+1)(n+2)$$
$$= \tfrac{1}{3}n(n+1)(n+2) + (n+1)(n+2)$$
$$= \tfrac{1}{3}n(n+1)(n+2)(n+3)$$

Thus if true for n then true for $n + 1$.

Therefore since true for $n = 1$, true for all $n \geq 1$. **[5]**

A8. $f(x) = \dfrac{2x^3 - 7x^2 + 4x + 5}{(x-2)^2}$

(*a*) $x = 0 \Rightarrow y = \frac{5}{4} \Rightarrow a = \frac{5}{4}$ **[1]**

(*b*) (i) $x = 2$ **[1]**

(ii) After division, the function can be expressed in quotient/remainder form:

$$f(x) = 2x + 1 + \frac{1}{(x-2)^2}$$

Thus the line $y = 2x + 1$ is a slant asymptote. **[3]**

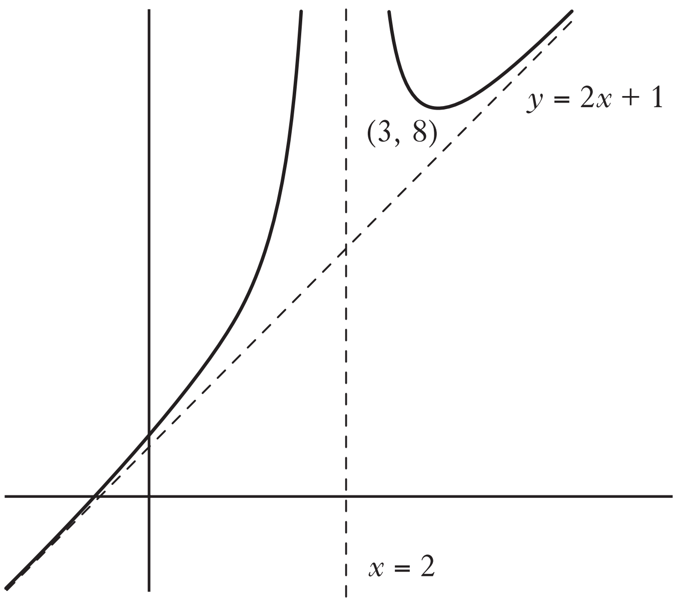
(*c*) From (*b*), $f'(x) = 2 - \dfrac{2}{(x-2)^3}$. Turning point when

$$2 - \frac{2}{(x-2)^3} = 0$$
$$(x-2)^3 = 1$$
$$x - 2 = 1 \Rightarrow x = 3$$
$$f''(x) = \frac{6}{(x-2)^4} > 0 \text{ for all } x.$$

The stationary point at (3, 8) is a minimum turning point. **[4]**

(*d*) $f(-2) = \dfrac{-16 - 28 - 8 + 5}{(-4)^2} < 0$; $f(0) = \dfrac{5}{4} > 0$.
Hence a root between –2 and 0. **[1]**

(*e*)



[2]

Marks

A9. (*a*) $z^4 = (\cos\theta + i\sin\theta)^4$

$= \cos^4\theta + 4\cos^3\theta\,(i\sin\theta) + 6\cos^2\theta\,(i\sin\theta)^2 + 4\cos\theta\,(i\sin\theta)^3 + (i\sin\theta)^4$

$= \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta + i\,(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)$

Hence the real part is $\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$.

The imaginary part is $(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)$

$= 4\cos\theta\sin\theta\,(\cos^2\theta - \sin^2\theta)$ **[5]**

(*b*) $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$ **[1]**

(*c*) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$ **[1]**

(*d*) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta\,(1 - \cos^2\theta) + (1 - \cos^2\theta)^2$

$= \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + 6\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

$= 8\,(\cos^4\theta - \cos^2\theta) + 1$

ie $k = 8$, $m = 4$, $n = 2$, $p = 1$. **[4]**

A10. (*a*) $900 = A(15 - Q) + B(30 - Q)$
Letting $Q = 30$ gives $A = -60$
and $Q = 15$ gives $B = 60$

$$\frac{900}{(30-Q)(15-Q)} = \frac{-60}{(30-Q)} + \frac{60}{(15-Q)}$$ **[2]**

(*b*) $$\frac{dQ}{dt} = \frac{(30-Q)(15-Q)}{900}$$

$$\therefore \int \frac{900}{(30-Q)(15-Q)}\,dQ = \int dt$$

$$\therefore \int \frac{-60}{(30-Q)} + \frac{60}{(15-Q)}\,dQ = \int dt$$

$$60\ln(30-Q) - 60\ln(15-Q) = t + C$$

ie $60\ln\left(\frac{30-Q}{15-Q}\right) = t + C$

$A = 60$

$C = 60\ln 2 = 41\cdot59$ to 2 decimal places **[4]**

(i) $$t = 60\ln\left(\frac{30-Q}{15-Q}\right) - 60\ln 2 = 60\ln\left(\frac{30-Q}{2(15-Q)}\right)$$

When $Q = 5$, $t = 60\ln\frac{25}{20} = 13\cdot39$ minutes to 2 decimal places. **[1]**

(ii) $$\ln\left(\frac{30-Q}{2(15-Q)}\right) = \frac{t}{60}$$

$$30 - Q = 2(15-Q)e^{t/60}$$

$$Q(2e^{t/60} - 1) = 30(e^{t/60} - 1)$$

$$Q = \frac{30(e^{t/60}-1)}{2e^{t/60}-1}$$

When $t = 45, Q = 10\cdot36$ grams to 2 decimal places. **[2]**

Section B (Mathematics 3)

B11.

$$\begin{aligned} 239 &= 1 \times 195 + 44 \\ 195 &= 4 \times 44 + 19 \\ 44 &= 2 \times 19 + 6 \\ 19 &= 3 \times 6 + 1 \\ \text{So } 1 &= 19 - 3 \times 6 \\ &= 19 - 3(44 - 2 \times 19) \\ &= 7 \times (195 - 4 \times 44) - 3 \times 44 \\ &= 7 \times 195 - 31(239 - 195) \\ &= 38 \times 195 - 31 \times 239 \end{aligned}$$

ie $195x + 239y = 1$ when $x = 38$ and $y = -31$ **[5]**

B12.

$$\begin{aligned} A^2 &= 5A + 3I \\ \therefore A^2 - 5A &= 3I \\ A(\tfrac{1}{3}A - \tfrac{5}{3}I) &= I \end{aligned}$$

$$\begin{aligned} A^4 &= (5A + 3I)^2 \\ &= 25A^2 + 30A + 9I \\ &= 155A + 84I \end{aligned}$$

$\therefore A$ is invertible and $A^{-1} = \frac{1}{3}(A - 5I)$ **[2, 2]**

B13. (i)

$$\begin{aligned} f(x) &= \sqrt{1+x} & f(0) &= 1 \\ &= (1+x)^{1/2} \\ f'(x) &= \frac{1}{2}(1+x)^{-1/2} & f'(0) &= \frac{1}{2} \\ f''(x) &= -\frac{1}{4}(1+x)^{-3/2} & f''(0) &= -\frac{1}{4} \\ f'''(x) &= \frac{3}{8}(1+x)^{-5/2} & f'''(0) &= \frac{3}{8} \end{aligned}$$

$\therefore \sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$ **[3]**

(ii)

$$\begin{aligned} f(x) &= (1-x)^{-2} & f(0) &= 1 \\ f'(x) &= 2(1-x)^{-3} & f'(0) &= 2 \\ f''(x) &= 6(1-x)^{-4} & f''(0) &= 6 \\ f'''(x) &= 24(1-x)^{-5} & f'''(0) &= 24 \end{aligned}$$

$\therefore (1-x)^{-2} \approx 1 + 2x + 3x^2 + 4x^3$ **[2]**

Marks

B14. $$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = f(x)$$

A.E. $m^2 - 5m + 6 = 0$

$\therefore m = 2$ or $m = 3$

C.F. $y = Ae^{2x} + Be^{3x}$

(i) $f(x) = 20\cos x$; P.I. $= a\cos x + b\sin x$

$\Rightarrow -a\cos x - b\sin x + 5a\sin x - 5b\cos x + 6a\cos x + 6b\sin x = 20\cos x$

$5a - 5b = 20$

$5a + 5b = 0 \Rightarrow a = -b$

$-10b = 20 \Rightarrow b = -2;\ a = 2$

Solution $y = Ae^{2x} + Be^{3x} + 2\cos x - 2\sin x$ **[3]**

(ii) $f(x) = 20\sin x$; P.I. $= c\cos x + d\sin x$

$5c - 5d = 0 \Rightarrow c = d$

$5c + 5d = 20 \Rightarrow c = d = 2$

Solution $y = Ae^{2x} + Be^{3x} + 2\cos x + 2\sin x$ **[3]**

(iii) $f(x) = 20\cos x + 20\sin x$

Solution $y = Ae^{2x} + Be^{3x} + 4\cos x$ **[1]**

B15. (*a*) L_1: $x = 3 + 2s$; $y = -1 + 3s$; $z = 6 + s$

L_2: $x = 3 - t$; $y = 6 + 2t$; $z = 11 + 2t$

$\therefore$ for x: $3 + 2s = 3 - t \Rightarrow t = -2s$

$\therefore$ for y: $3s - 1 = 6 + 2t$

$7\lambda = 7 \Rightarrow s = 1;\ t = -2$

$\therefore L_1$: $x = 5$; $y = 2$; $z = 6 + s = 7$

$\therefore L_2$: $x = 5$; $y = 2$; $z = 11 + 2t = 11 - 4 = 7$

ie L_1 and L_2 intersect at (5, 2, 7) **[6]**

(*b*) $A(2,1,0)$; $B(3,3,-1)$; $C(5,0,2)$

$\overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$; $\overrightarrow{AC} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$$

Equation of plane has form $3x - 5y - 7z = k$

$(2, 1, 0) \Rightarrow k = 1$

Equation is $3x - 5y - 7z = 1$. **[5]**

Marks

Section C (Statistics 1)

C11. $P(A) = 0\cdot65 \quad P(\text{def}|A) = 0\cdot02$ Bayes Th. $P(A|\text{def}) = \dfrac{0\cdot02 \times 0\cdot65}{0\cdot02 \times 0\cdot65 + 0\cdot05 \times 0\cdot35} = \dfrac{26}{61}$

$P(B) = 0\cdot35 \quad P(\text{def}|B) = 0\cdot05$ **[5]**

C12. $P(X > 8) = 1 - p(X \le 8) = 0\cdot407$ **[2]**

$\left.\begin{array}{l} P(X \le 12) = 0\cdot936 \\ P(X \le 13) = 0\cdot966 \end{array}\right\}$ 13 required **[2]**

C13. $B\,(100, \frac{1}{2}) \quad P(B \ge 55) \approx P\left(Z \ge \dfrac{55 - \frac{1}{2} - 50}{5}\right) = 0\cdot184$ **[4]**

C14. Take a random sample of all schools with Higher Mathematics candidates and then select every 5th year pupil, taking Higher Mathematics, from those schools. **[2]**

A list of all Higher Mathematics candidates would not be available to us and there would be a high cost in terms of time and money if we had to visit widely scattered schools. **[2]**

C15. $\hat{p} = 0\cdot375$

$\hat{q} = 0\cdot625$ 95% C.I. is $0\cdot375 \pm 1\cdot96\sqrt{\dfrac{0\cdot375 \times 0\cdot625}{40}}$

$n = 40$ $= 0\cdot225 \rightarrow 0\cdot525$

For every one hundred intervals calculated we would expect 95 of them to capture the true value of p and 5 not to.

We must assume that the 40 fish constitute a random sample. **[6]**

C16. $\overline{X} \sim N\left(21, \left(\dfrac{4}{\sqrt{10}}\right)^2\right) \qquad P(\overline{X} < 17\cdot7) = P\left(Z < \dfrac{17\cdot7 - 21}{\frac{4}{\sqrt{10}}}\right) = 0\cdot005$ **[4]**

$\overline{X} = 17\cdot7 \qquad H_0 : \mu = 21 \qquad H_1 : \mu < 21 \qquad 1 - \text{tail test}$

$P(\overline{X} \le 17\cdot7) = 0\cdot005 < 0\cdot01$

Reject H_0 at 1% level ie there is strong evidence of a reduction in waiting time. **[5]**

Section D (Numerical Analysis 1)

D11. $f(x) = \ln(3x-2); \quad f'(x) = \dfrac{3}{3x-2}; \quad f''(x) = \dfrac{-9}{(3x-2)^2}$

$$f'''(x) = \frac{54}{(3x-2)^3}$$

2nd degree polynomial is

$p_2(x) = p_2(1+h) = f(1) + hf'(1) + \dfrac{h^2}{2}f''(1)$

$= \ln 1 + 3h - 4\cdot5h^2 = 3h - 4\cdot5h^2$ **[3]**

At $x = 1\cdot05,\ h = 0\cdot05$ so $f(1\cdot05) \approx 0\cdot1388$ **[2]**

Principal error $= \dfrac{h^3}{3!}\dfrac{54}{(3-2)^3} = 0\cdot0011$

Actual error $= 0\cdot1388 - \ln 1\cdot15 = 0\cdot00096$

Values are equal to 3 decimal places **[2]**

D12. $L(x) = \dfrac{(x-2)(x-4)}{(-1)(-3)}2\cdot7483 + \dfrac{(x-1)(x-4)}{1(-2)}2\cdot3416 + \dfrac{(x-1)(x-2)}{3.2}1\cdot2268$

$= (x^2-6x+8)\dfrac{2\cdot7483}{3} - (x^2-5x+4)\dfrac{2\cdot3416}{2} + (x^2-3x+2)\dfrac{1\cdot2268}{6}$

$= -0\cdot0502x^2 - 0\cdot2560 + 3\cdot0545$ **[4]**

D13. Consider the quadratic through $(x_0, f_0), (x_1, f_1), (x_2, f_2)$

Let equation be $y = A_0 + A_1(x-x_0) + A_2(x-x_0)(x-x_1)$

Then $f_0 = A_0; \quad f_1 = A_0 + A_1h; \quad A_2 = \dfrac{f_2 - 2f_1 + f_0}{2h^2} = \dfrac{\Delta^2 f_0}{2h^2}$

Thus $y = f_0 + \dfrac{x-x_0}{h}\Delta f_0 + \dfrac{(x-x_0)(x-x_1)}{2h^2}\Delta^2 f_0$

Setting $x = x_0 + ph$, when $0 < p < 1$, gives

$$y = f_0 + p\Delta f_0 + \frac{p(p-1)}{2}\Delta^2 f_0$$ **[5]**

x_i	f_i	Δf_i	$\Delta^2 f_i$
1·6	0·826		
		377	
1·7	1·203		29
		406	
1·8	1·609		27
		433	
1·9	2·042		28
		461	
2·0	2·503		

$p = \dfrac{0\cdot03}{0\cdot1} = 0\cdot3$

$f(1\cdot63) \approx 0\cdot826 + 0\cdot3 \times 0\cdot377 + \dfrac{0\cdot3.(-0\cdot7)}{2}0\cdot029 = 0\cdot936$ **[4]**

D14.

x	$f(x)$	m	$mf(x)_4$	m	$mf(x)_2$
1·00	0·13534	1	0·13534	1	0·13534
1·25	0·20040	4	0·80160		
1·50	0·25205	2	0·50410	4	1·00820
1·75	0·28322	4	1·13288		
2·00	0·29305	1	0·29305	1	0·29305
			2·86697		1·43659

$I_1 = \frac{1 \cdot 43659}{6} = 0 \cdot 23943$ $\quad$ $I_2 = \frac{2 \cdot 86697}{12} = 0 \cdot 23891$ **[4]**

$|E| \leq \frac{1}{180} \times 0 \cdot 25^4 \times 1 \cdot 903 = 4 \cdot 13 \times 10^{-5} = 0 \cdot 000041$

Hence $I_2 = 0 \cdot 2389$. **[2]**

With n strips and width $2h$, the Taylor series for an integral approximated by Simpson's rule, with principal truncation error $O(h^4)$, is

$I = I_n + C(2h)^4 + D(2h)^6 + \ldots. \quad = I_n + 16Ch^4 + \ldots. \qquad -(1)$

Similarly, with $2n$ strips of width h,

$I = I_{2n} + Ch^4 + Dh^6 + \ldots \quad -(2)$

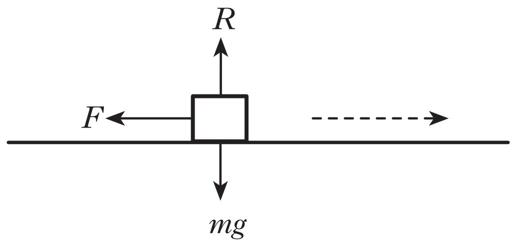
Taking $16 \times (2) - (1)$ gives $15I = 16I_{2n} - I_n + O(h^6)$

$\Rightarrow I = I_{2n} + \frac{1}{15}(I_{2n} - I_n)$ **[4]**

and $I_3 = 0 \cdot 23891 + \frac{1}{15}(-0 \cdot 00052) = 0 \cdot 23888$ **[2]**

Section E (Mechanics 1)

E11.



$R = mg$

$ma = -F = -\mu R$

$ma = -0\cdot04mg$

$a = -0\cdot04g$

$v^2 = u^2 + 2as$

$0 = u^2 - 2 \times 0\cdot04g \times 28$

$u^2 = 21\cdot952 \Rightarrow u = 4\cdot7 \text{ m s}^{-1}$ [to 1 decimal place] **[4]**

E12. $a = \frac{1}{3}(13 - 2t)$

$v = \frac{1}{3}(13t - t^2) + c$

$v = 12 \quad t = 0 \Rightarrow c = 12 \Rightarrow v = \frac{1}{3}(13t - t^2) + 12$

$\frac{1}{3}(13t - t^2) + 12 = 26 \Rightarrow t^2 - 13t + 42 = 0$

$(t-6)(t-7) = 0 \quad t = 6,7$

First reaches 26 m s^{-1} after 6 secs.

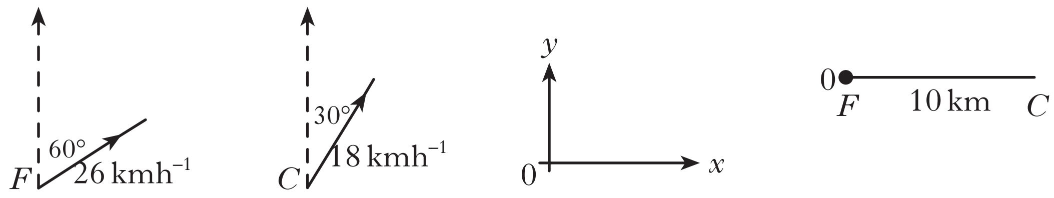
$s = \frac{1}{3}\left(\frac{13t^2}{2} - \frac{t^3}{3}\right) + 12t + c$

$s = -40 \quad t = 0 \Rightarrow c = -40$

$s = \frac{13t^2}{6} - \frac{t^3}{9} + 12t - 40$

$t = 6 \quad s = 78 - 24 + 72 - 40 = 86 \text{ m}$ outside the built-up area. **[5]**

E13.



$\mathbf{v}_C = 18\sin 30\,\mathbf{i} + 18\cos 30\,\mathbf{j} = 9\mathbf{i} + 15\cdot 6\,\mathbf{j}$

$\mathbf{r}_F = 22\cdot 5t\mathbf{i} + 13t\mathbf{j}$ $\qquad\qquad$ $\mathbf{r}_c = (9t+10)\mathbf{i} + 15\cdot 6t\,\mathbf{j}$

$\Rightarrow \mathbf{r}_F - \mathbf{r}_C = (13\cdot 5t - 10)\mathbf{i} - 2\cdot 6t\mathbf{j}$

$\left|\mathbf{r}_F - \mathbf{r}_C\right|^2 = (13\cdot 5t - 10)^2 + 6\cdot 76t^2$

$\dfrac{d\left|\mathbf{r}_F - \mathbf{r}_C\right|^2}{dt} = 27(13\cdot 5t - 10) + 13\cdot 5t = 0$ for stationary point

$\Rightarrow 378t = 270$

$t = 0\cdot 71\text{ h}$

$= 43\text{min}$

Vessels will be closest at 12.43 pm **[7]**

E14. $R = \dfrac{u^2\sin 2\alpha}{g} \Rightarrow$ Max range $= \dfrac{u^2}{g} = 60 \Rightarrow u^2 = 60\text{g}$

$u = 24\cdot 2\text{ m s}^{-1}$

Max height when $0 = u^2\sin^2\alpha - 2gh$

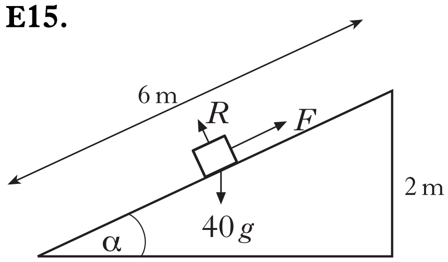
$0 = 60\text{ g} \times \dfrac{1}{2} - 2gh$

$h = \dfrac{30g}{2g} = 15\text{ m}$

So max height above ground $= 16\cdot 5$ m **[6]**

Marks

E15.



Perp to plane $R = 40g\cos\alpha$

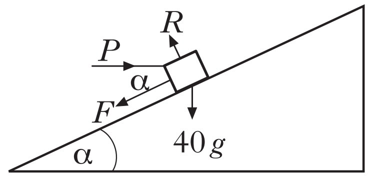
Parallel to plane $F = 40g\sin\alpha$

On point of slipping $F = \mu R$

$\Rightarrow \mu 40g\cos\alpha = 40g\sin\alpha$

$\mu = \tan\alpha = \frac{2}{\sqrt{32}} = \frac{2}{4\sqrt{2}} = \frac{\sqrt{2}}{4}$ (or 3·5)

[3]



Perp to plane $R = 40g\cos\alpha + P\sin\alpha$

Parallel to plane $F + 40g\sin\alpha = P\cos\alpha$

$F = \frac{\sqrt{2}}{4}R \Rightarrow \frac{\sqrt{2}}{4}\left(40g\frac{\sqrt{32}}{6} + P\frac{1}{3}\right) + 40g.\frac{1}{3} = P\frac{\sqrt{32}}{6}$

$\frac{40g}{3} + \frac{P\sqrt{2}}{12} + \frac{40g}{3} = \frac{2\sqrt{2}P}{3}$

(candidates likely to use μ = 0·35 and numerical value of α)

$80g = (2 - \frac{1}{4})\sqrt{2}P$

$P = \frac{80 \times 9\cdot 8}{1\cdot 75 \times \sqrt{2}} = 316\cdot 8\ \text{N}$ **[7]**

[*END OF MARKING INSTRUCTIONS*]